

## Deterministic and stochastic oscillations of fractal type during cooling of the melt

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**Резюме.** A "single-phase" model of melt crystallization in the Penrose–Fife representation for temperature distributions under non-isothermal conditions is considered. The boundary conditions are assumed to be nonlinear and dynamic, i.e. they characterize the relaxation rate of the surface order parameter. In this case, the boundary conditions depend on the frequency of new phase nucleation and the rate of a melt crystallization in the (near-)surface layers of the mold. A method is proposed for predicting the appearance of ordered spatiotemporal (quasi-)crystalline structures of the fractal type of the crystalline phase in a liquid melt. The surface order parameter determines the mechanical and deformation properties of the sample depending on the temperature of the solid.

**Ключевые слова**

**amorphous melt, difference equation with quasi-periodic perturbations,  
period-doubling bifurcations**

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