УДК 532.5.031+550.344.42+551.466.62

https://doi.org/10.30730/gtrz.2025.0.wif-2;
https://elibrary.ru/loobrn
http://journal.imgg.ru/web/full/f-e2025-0-2.pdf (In English)
http://journal.imgg.ru/web/full/f2025-0-2.pdf (In Russian)

Waves in a fluid layer excited by pressure variations above the free surface

Yury P. Korolev

E-mail: Yu_P_K@mail.ru

Institute of Marine Geology and Geophysics, FEB RAS, Yuzhno-Sakhalinsk, Russia

Abstract. The aim of the paper was to study the problem of waves in a layer of incompressible fluid of constant depth. The interest in the problem arose due to the excitation and propagation of surface waves in the Pacific Ocean as a result of the powerful explosive eruption of the Hunga Tonga–Hunga Haapai volcano on January 15, 2022. Potential fluid motions were considered. The disturbances were induced in the form of a short-term pressure pulse above the free surface and in the form of pressure waves arising due to of the disintegration of the initial region of high pressure in the atmosphere (Lamb waves). Solutions were obtained for forced and free waves on the surface, as well as for forced and free pressure waves at the bottom of the fluid layer. In the long-wave approximation, the amplitudes of free surface waves and the amplitudes of free bottom pressure waves (in meters of water column) coincide, while the amplitudes of forced bottom pressure waves are greater than the amplitudes of forced surface waves. In cases where only the forced component is present in the pressure record, the use of a correction factor gives an adequate result for surface waves. If both components (forced and free) are present in the record, the use of the correction factor is unjustified, since it is impossible to separate the components. The estimation of surface wave amplitudes based on bottom pressure data may yield inadequate results. The results obtained are discussed in connection with the operational tsunami forecast based on the data from bottom sea level measurement stations. A proposal is formulated on a possible method for adequately estimating the amplitude of surface waves when excited by a moving region of variable pressure.

Keywords: water waves, Lamb waves, forced waves, baric waves, free waves, gravity waves, tsunami, sea level measurements, operational tsunami forecast, tsunami warning services, Pacific Ocean

Волны в слое жидкости, возбуждаемые вариациями давления над свободной поверхностью

Ю. П. Королёв

E-mail: Yu_P_K@mail.ru

Институт морской геологии и геофизики ДВО РАН, г. Южно-Сахалинск, Россия

Резюме. Целью работы являлось исследование задачи о волнах в слое несжимаемой жидкости постоянной глубины. Интерес к задаче возник в связи с возбуждением и распространением поверхностных волн в Тихом океане в результате мощного эксплозивного извержения вулкана Хунга Тонга–Хунга Хаапай 15.01.2022. Рассматривались потенциальные движения жидкости. Возмущения задавались в виде кратковременного импульса давления над свободной поверхностью и в виде волн давления, возникающих в результате распада начальной области повышенного давления в атмосфере (волн Лэмба). Получены решения для вынужденных и свободных волн на поверхности, а также вынужденных и свободных волн давления на дне слоя жидкости. В приближении длинных волн амплитуды свободных поверхностных волн и амплитуды свободных волн

The translation from Russian: Королев Ю.П. Волны в слое жидкости, возбуждаемые вариациями давления над свободной поверхностью. http://journal.imgg.ru/web/full/f2025-0-2.pdf. *Translated by Yury P. Korolev.*

придонного давления (в метрах водяного столба) совпадают, в то время как амплитуды вынужденных волн придонного давления выше амплитуд вынужденных поверхностных волн. В случаях, когда в записи давления присутствует только вынужденная составляющая, применение корректирующего множителя дает адекватный результат для поверхностных волн. Если в записи присутствуют обе компоненты (вынужденная и свободная), применение поправочного коэффициента неправомерно, поскольку разделить составляющие невозможно. Оценка амплитуд поверхностных волн по данным о давлении на дне может давать неадекватный результат. Полученные результаты обсуждаются в связи с оперативным прогнозом цунами по данным донных станций измерения уровня океана. Сформулировано предложение о возможном способе адекватной оценки амплитуды поверхностных волн при возбуждении их движущейся областью переменного давления.

Ключевые слова: волны на воде, волны Лэмба, вынужденные волны, барические волны, свободные волны, гравитационные волны, цунами, измерения уровня океана, оперативный прогноз цунами, службы предупреждения о цунами, Тихий океан

For citation: Korolev Yu.P. Waves in a fluid layer excited by pressure variations above the free surface. *Geosistemy perehodnykh zon = Geosystems of Transition Zones*, 2025. https://doi.org/10.30730/gtrz.2025.0.wif-2; https://elibrary.ru/loobm; http://journal.imgg.ru/web/full/f-e2025-0-2.pdf

Для цитирования: Королёв Ю.П. Волны в слое жидкости, возбуждаемые вариациями давления над свободной поверхностью. *Геосистемы переходных зон*, 2025. https://doi.org/10.30730/gtrz.2025.0.wif-2; https://elibrary.ru/loobm; http://journal.imgg.ru/web/full/f2025-0-2.pdf

Introduction

The powerful explosive eruption of the Hunga Tonga–Hunga Ha'apai volcano in the South Pacific Ocean on January 15, 2022[1], is estimated to be the largest underwater volcanic eruption in almost a century and a half since the catastrophic destruction of Krakatoa in 1883 [1]. The effects of the explosion were observed in all environments: the ionosphere, the atmosphere, the ocean and its surface, and the earth's crust [2–6].

The tsunami generated by the volcanic eruption caused catastrophic floods the nearby islands of the Tonga archipelago with a maximum height of up to 22 m. The tsunami caused damage not only to the nearby island states, but also to the countries of the Pacific coast. Flood heights of up to 1.3 m were recorded in Japan, over 3.5 m in California, about 1 m in Chile and up to 1 m in Peru (https://www.ngdc.noaa.gov/hazel/view/hazards/tsunami/event-search).

The volcanic explosion produced a high-pressure wave in the atmosphere (also called a Lamb wave), which was recorded by many ground-based barographs around the globe. The high-pressure wave, propagating at a speed close to the speed of sound in the atmosphere, caused disturbances in the free surface of the ocean in the form of a forced wave moving at the same speed. Such a wave is called a baric wave below. A rapid (explosive) change in atmospheric pressure is itself a source of gravity waves on the water surface. Other processes in the eruption center, leading to changes in the water surface level, are also sources of gravity (free) waves in the ocean, propagating at the speed of long waves. In any case, surface waves are a superposition of baric and gravity waves after the latter arrive at the observation point. Baric and/or gravity waves have been recorded by many deep-sea bottom stations of the DART (Deep-ocean Assessment and Reporting of Tsunamis)[2] system in the Pacific Ocean[3]. Both waves, baric and gravity, have been recorded in their entirety, from the moment of arrival of the baric wave, by a small number of DART stations. The amplitudes of the baric and gravity waves are comparable even at large distances from the eruption. The change in the amplitude of baric waves is inversely proportional to the square root of the distance from the source [7],

[1] NOAA National Centers for Environmental Information. URL: https://www.ngdc.noaa.gov/hazel/view/hazards/tsunami/event-search (accessed 08.06.2025).

[2] NOAA Center for Tsunami Research: DART. URL: http://nctr.pmel.noaa.gov/Dart (accessed 08.06.2025).

[3] National Data Buoy Center. URL: https://ndbc.noaa.gov/to_station.shtml (accessed 08.06.2025).

as is the change in the amplitude of gravity waves. Waves of this type, cylindrical waves, described in the space of two variables, have a leading edge but no trailing edge, and the oscillations behind the front continue for quite a long time [8]. Although the gravity wave lags noticeably behind the baric wave, with its arrival a superposition of the gravity and barometric waves occurs.

Various phenomena in the atmosphere, on the surface and on the ocean floor that accompanied the volcanic explosion are considered in numerous works. In work [9], the processes that occurred in the source were discussed. It was assumed that five explosions of varying intensity occurred in the area of the volcano within half an hour to an hour. Pressure waves in the atmosphere based on natural data were analyzed in works [3, 10]. In article [7], data from many ground-based barographs were analyzed, it was established that the pressure wave in the atmosphere (Lamb wave) propagated at a speed of 317 m/s, its amplitude decreased with distance from the explosion as $r^{-1/2}$, and numerical modeling of pressure waves was performed based on a specially constructed source. A close estimate of the propagation speed of the Lamb wave of 312 m/s was obtained in [11]. The influence of atmospheric pressure waves on the generation of waves on the ocean surface based on numerical modeling is considered in [4, 7, 12–17], as well as on the website of the NOAA Center for Tsunami Research[4]. In [4], differences in the amplitudes of bottom pressure waves and surface waves were noted. The generation of gravity waves as a result of disturbances of the water surface at the source in a numerical model is considered in [7, 18, 19]. The parameters of the disturbance sources were selected based on the degree of coincidence of the shapes of the computed and recorded waves in the ocean.

Most of the listed works devoted to the event on January 15, 2022 were the result of either numerical experiments or analysis of processes in the source.

Waves from a moving region of increased atmospheric pressure in the “shallow water” approximation were considered in [20]. When such regions propagate at a speed significantly lower than the speed of long waves in the open ocean, the Proudman resonance can only occur in shallow water, when the speed of long waves approaches the speed of the baric disturbance [21]. In contrast, Lamb waves propagate at a speed close to the speed of sound in air. Resonance can occur in areas of deep-water, but rather narrow trenches, the speed of long waves over which exceeds the speed of the Lamb wave. For example, the Mariana Trench, with a maximum depth of about 11 km, has an average width of 69 km (https://en.wikipedia.org/wiki/Mariana_Trench). The influence of such depressions on the propagation of free and forced waves on the ocean surface has not been studied.

In [22], solutions are given to problems of excitation of surface waves from short-term pressure action on a free surface, and of excitation of waves by a moving area of increased pressure, which can result in the occurrence of meteotsunamis. The forms of surface waves and bottom pressure waves were analyzed. The differences between variations in bottom pressure and variations in the free surface were discussed.

The problem of waves excited by a high-pressure region moving at a constant velocity in a one-dimensional formulation is considered in [6, 11, 23], and in a two-dimensional formulation in [23]. The moving region excites forced waves on the free surface and on the bottom. It is shown that the amplitudes of the bottom pressure waves are greater than the amplitudes of the forcing pressure above the free surface, the amplitudes of the bottom pressure waves, expressed in meters of water column, are higher than the amplitudes of the surface waves. The works [17, 23] describe the correction of data on bottom pressure variations for estimating the shape of forced waves. Estimates of free waves were not considered.

[4] NOAA Center for Tsunami Research: Volcano-generated Tsunami Event – January 15, 2022 Hunga Tonga–Hunga Ha’apa Tsunami. URL: https://nctr.pmel.noaa.gov/tonga20220115/ (accessed 08.06.2025).

Data on the bottom pressure of the deep-ocean stations closest to the source are used in operational tsunami forecasting by the current NOAA method[5], the express method [24], to estimate the waveform of the expected tsunami at more remote points and near the coast. As shown on the NOAA Center for Tsunami Research website, overestimated amplitudes of surface wave based on bottom pressure data can be the cause of an inadequate determination of the degree of danger of the predicted tsunami.

The objective of the work is to study the solution of the problem of waves in a liquid layer generated by atmospheric pressure disturbances. Waves on the liquid surface and bottom pressure waves arising under the action of traveling atmospheric pressure waves (Lamb waves) are considered. The problem is of interest in connection with the use of ocean bottom pressure data in operational tsunami forecasting.

Statement of the problem

We consider the classical problem of potential motion in a layer of heavy liquid of depth H lying on a solid foundation [8]. The problem is solved in the space of three variables, the Oz axis with the origin on the free surface is directed vertically upward, the Ox and Oy axes are on the free surface. The acceleration of gravity g is directed downward.

The velocity potential φ in the liquid layer satisfies the equation

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}=0\,. \qquad (1)$$

For liquids, the Bernoulli equation is valid

$$\frac{d^2\varphi}{dz^2}-\xi^2\varphi=0,$$

where ρ is the density of the liquid (water), p_a is the atmospheric pressure, p is the pressure in the liquid layer.

In the linear approximation, the boundary conditions for (1) are

on the disturbed free surface $z = \zeta$ (the values of the variables φ and p are related to $z = 0$):

$$\frac{\partial\zeta}{\partial t}=\frac{\partial\varphi}{\partial z} \qquad (2)$$

and

$$\frac{\partial\varphi}{\partial t}+\frac{p}{\rho}+g\zeta=\frac{p_a}{\rho}; \qquad (3)$$

at the bottom ($z = –H$) $\frac{\partial\varphi}{\partial z}=0$. (4)

In this paper, the disturbance is specified by the pressure above the free surface $p = p_0 + p_a$, including some axially symmetric disturbance $p_0(r,t)$. Also, for generality, an axially symmetric initial elevation of the free surface can be specified, caused by another possible generation mechanism, not necessarily associated with pressure changes. Due to the linearity of the problem, the waves excited by these sources do not affect each other.

The pressure at the bottom ($z = –H$) is determined from the Bernoulli equation using the found φ:

$$p=-\rho\frac{\partial\varphi}{\partial t}+p_a+\rho gH=p_{bott}+p_a+\rho gH, \qquad (5)$$

where p_{bott} are the variations in pressure at the bottom.

Problem (1)–(4) in cylindrical coordinates is solved by one of the common methods – by using the integral Laplace transform with respect to time ($f(s)=\int_0^\infty f(t)\cdot e^{-st}dt$) and the Fourier–Bessel transform ($f(\xi)=\int_0^\infty f(r)\cdot J_0(\xi r)rdr$) with respect to the radial coordinate [25].

The solution is presented in the form of Fourier–Bessel transform images. This is sufficient for a comparative analysis of waves on a free surface and waves on the bottom.

[5] NOAA Center for Tsunami Research: Tsunami Forecasting. URL: https://nctr.pmel.noaa.gov/tsunami-forecast.html (accessed 08.06.2025).

Results and discussion

Waves excited by a short-term pressure pulse above the free surface

The initial condition for problem (1)–(4) is the elevation of the free surface $\zeta(t=0,r)=\zeta_0(r/R_1)$ inside the circle $r<R_1$, $(r^2=x^2+y^2)$. The pressure disturbance above the free surface is given by a short-term axially symmetric pressure pulse in the circle $r<R$: $p_0=p_0(r/R)\,\delta(t/T)$, where $\delta(t/T)$ is the Dirac δ-function.

After integral transformations, system (1)–(4) is represented as

$$\frac{d^2\varphi}{dz^2}-\xi^2\varphi=0. \qquad (1.1)$$

Boundary conditions:
at $z=0$

$$s\zeta-Z_0=\frac{d\varphi}{dz}, \qquad (2.1)$$

$$s\varphi+\frac{P_0T}{\rho}+g\zeta=0; \qquad (3.1)$$

at $z=-H$

$$\frac{d\varphi}{dz}=0. \qquad (4.1)$$

Initial condition:

$$Z_0=\int_0^{R_1}\zeta_0(r/R_1)\cdot J_0(\xi r)\cdot r dr,\ \varphi(t=0)=0.$$

P_0T in (3.1) is defined as

$$P_0T=\int_0^R\int_0^\infty p_0(r/R)\cdot\delta(t/T)\cdot e^{-st}\cdot J_0(\xi r)\cdot dt\cdot r dr=$$

$$=T\int_0^R p_0(r/R)\cdot J_0(\xi r)\cdot r dr.$$

In the system (1.1)–(4.1) all variables are images of integral transformations. Below, the arguments of the function (s and/or ξ) indicate the image of which transformation this function is.

The solution to (1.1) is $\varphi=A_1e^{\xi z}+A_2e^{-\xi z}$. The unknown coefficients A_1 and A_2, as well as $\zeta(s,\xi)$ are found from the solution of the system (2.1)–(4.1) taking into account the pressure disturbance and the initial elevation of the free surface.

$$\varphi(z,s,\xi)=-\frac{P_0T}{\rho}\frac{ch\xi(z+H)}{ch\xi H}\frac{s}{s^2+\Omega^2}-$$

$$-gZ_0\frac{ch\xi(z+H)}{ch\xi H}\frac{1}{s^2+\Omega^2};$$

$$\zeta(s,\xi)=-\frac{P_0T}{\rho g}\frac{\Omega^2}{s^2+\Omega^2}+Z_0\frac{s}{s^2+\Omega^2}.$$

The variations in pressure at the bottom are found from (5):

$$p_{bott}(z=-H,s,\xi)=\frac{P_0T}{ch\xi H}\frac{s^2}{s^2+\Omega^2}+\frac{\rho g Z_0}{ch\xi H}\frac{s}{s^2+\Omega^2}=$$

$$=\frac{P_0T}{ch\xi H}\left(1-\frac{\Omega^2}{s^2+\Omega^2}\right)+\frac{\rho g Z_0}{ch\xi H}\frac{s}{s^2+\Omega^2},$$

where $\Omega^2=g\xi\cdot th\xi H$.

The presented expressions have 2 poles: $s=\pm i\Omega$, corresponding to free waves.

The inverse Laplace transform gives:

$$\zeta(t,\xi)=-\frac{P_0}{\rho g}\Omega T\sin\Omega t+Z_0\cos\Omega t,$$

$$p_{bott}(-H,t,\xi)=\frac{P_0}{ch\xi H}\delta(t/T)-$$

$$-\frac{P_0}{ch\xi H}\Omega T\sin\Omega t+\frac{\rho g Z_0}{ch\xi H}\cos\Omega t.$$

The solutions coincide with the result obtained in a slightly different way in [22]. The wave components coincide with an accuracy of a dimensional factor.

In the approximation of long waves ($\xi H<<1$, $ch\xi H\approx 1$, $\Omega^2\approx gH\xi^2$) the obtained solutions are transformed into the following expressions.

Wave form of the free surface:

$$\zeta(t,\xi)=-\frac{P_0T}{\rho g}\sqrt{gH}\,\xi\sin\sqrt{gH}\,t\xi+Z_0\cos\sqrt{gH}\,t\xi,$$

where the first term describes the wave from the pressure pulse, the second one – the wave from the initial elevation of the free surface.

Pressure variations at the bottom, recorded by bottom stations, reduced to meters of water column:

$$\eta_{bott}(-H,t,\xi)=\frac{p_{bott}(-H,t,\xi)}{\rho g}=\frac{P_0}{\rho g}\cdot\delta(t/T)-$$
$$-\frac{P_0T}{\rho g}\cdot\sqrt{gH}\,\xi\sin\sqrt{gH}t\xi+Z_0\cos\sqrt{gH}t\xi\,.$$

Here the first term describes the reaction of the bottom pressure to the pressure impulse above the free surface, the second and third are similar to the terms from the previous expression.

In the obtained expressions, the wave components are identical. This allows us to quite reasonably estimate the shape (its long-wave component) of the ocean surface based on the bottom pressure data obtained by deep-sea stations in the ocean (https://ndbc.noaa.gov/to_station.shtml).

Waves generated by a diverging concentric wave of increased pressure above the free surface

Of interest is the problem of waves in a liquid generated by a traveling wave of increased pressure above the free surface, excited by the disintegration of an instantaneous increase in pressure in a limited circular region. According to observations, such a wave (Lamb wave) propagates at a speed close to the speed of sound in air, the amplitude attenuates with distance from the center of disturbance as $r^{-1/2}$ [1].

The formulation of the problem of waves on the surface of a liquid layer excited by such a wave coincides with the statement of problem (1)–(4), or (1.1)–(4.1) in the images of integral transformations. A wave in the atmosphere arises as a result of the disintegration of an initial region of high pressure in a circle $r < R$: $p_0 = p_0(r/R)$, which Fourier–Bessel image is $P_0=\int_0^R p_0(r/R)\cdot J_0(\xi r)\cdot rdr$. A wave from such a disturbance over a free surface in the images of integral transformations is described as $p_{\text{forcing}} = P_0 s/(s^2+U^2\xi^2)$. The wave front propagates with the velocity U, the amplitude asymptotically decays with distance as $r^{-1/2}$ [26]. The initial pressure pulse excites free waves, which front moves with the velocity of long waves. And the high-pressure wave gives rise to forced waves on the free surface, propagating with the velocity U. In equation (3.1), P_0T should be replaced by $P_0 s/(s^2+U^2\xi^2)$.

Besides, an additional initial condition is the elevation of the free surface $\zeta(t = 0,r) = \zeta_0(r/R_1)$ inside the circle $r < R_1$, $(r^2 = x^2 + y^2)$.

The solution to the problem in the images of integral transforms is represented by expressions for variations in the forcing pressure above the free surface $p_{forcing}(s,\xi) = P_0 s/(s^2 + U^2\xi^2)$, variations in the pressure at the bottom

$$p_{bott}(z=-H,s,\xi)=\frac{P_0}{ch\xi H}\frac{s}{s^2+U^2\xi^2}\frac{s^2}{s^2+\Omega^2}+$$

$$+\frac{\rho g Z_0}{ch\xi H}\frac{s}{s^2+\Omega^2}$$ and the shape of the free surface

$$\zeta(s,\xi)=-\frac{P_0}{\rho g}\frac{s}{s^2+U^2\xi^2}\frac{\Omega^2}{s^2+\Omega^2}+Z_0\frac{s}{s^2+\Omega^2}\cdot$$

The last two expressions have 4 poles: $s = \pm iU\xi$, corresponding to forced waves, and $s = \pm i\Omega$, corresponding to free waves.

The inverse Laplace transform in the long-wave approximation ($\xi H << 1$, $\Omega \approx (gH)^{1/2}\xi$, $\mathrm{ch}\xi H \approx 1$) yields the following expressions:

for the variations in the forcing pressure above the free surface (pressure is expressed in meters of water column)

$$\eta_{forcing}(t,\xi)=\frac{p_{forcing}(0,t,\xi)}{\rho g}=\frac{P_0}{\rho g}\cdot\cos Ut\xi, \quad (6)$$

for the variations in the bottom pressure (in meters of water column)

$$\eta_{bott}(-H,t,\xi)=\frac{p_{bott}(-H,t,\xi)}{\rho g}=\frac{P_0}{\rho g}\cdot\frac{U^2}{U^2-gH}\cos Ut\xi-$$
$$-\frac{P_0}{\rho g}\cdot\frac{gH}{U^2-gH}\cos\sqrt{gH}t\xi+Z_0\cos\sqrt{gH}t\xi \quad (7)$$

and for the shape of the free surface

$$\zeta(t,\xi) = \frac{P_0}{\rho g} \cdot \frac{gH}{U^2 - gH} \cos Ut\xi -$$

$$- \frac{P_0}{\rho g} \cdot \frac{gH}{U^2 - gH} \cos \sqrt{gH} t\xi + Z_0 \cos \sqrt{gH} t\xi. \quad (8)$$

In the obtained expressions (7) and (8), the first terms on the right-hand side describe forced waves propagating with the velocity U, the second and third terms describe free (gravity) waves excited by the initial pressure jump and the initial elevation of the free surface, moving with the velocity of long waves $(gH)^{1/2}$. To represent the expressions in spatial variables, the inverse Fourier–Bessel transform should be performed over them. For example, $\eta(t,r) = \frac{P_0}{\rho g} \int_0^\infty \cos Ut\xi \cdot J_0(r\xi)\xi d\xi$. Due to the known asymptotics of the Bessel function $J_0(r\xi) = \sqrt{\frac{2}{\pi r\xi}} \cos(r\xi - \frac{\pi}{4})$, the asymptotic estimate of the corresponding integrals will give the attenuation of the amplitudes as $r^{-1/2}$ [26].

From the comparison of (6)–(8) it is evident that the amplitude of the forced bottom pressure waves is greater than the amplitude of the forcing pressure by a factor of $U^2/(U^2–gH)$, the amplitude of the baric wave differs from the amplitude of the forcing pressure in meters of water column by a factor of $gH/(U^2–gH)$, and the amplitude of the forced bottom pressure waves is greater than the amplitude of the forced surface (baric) waves by a factor of U^2/gH. The result coincides with the conclusions of works [6, 11, 17, 23] described in the Introduction. The expressions describing the free waves are identical.

To estimate the variations in pressure above the free surface, the data from deep-sea bottom stations should be multiplied by the correction factor $(U^2–gH)/U^2$. This is confirmed in works [6, 11] based on the measurement data. To estimate the amplitude of forced surface (baric) waves based on bottom station data expressed in meters of water column, another correction factor gH/U^2 should be used [17]. At a velocity of $U = 317$ m/s [7] and an average ocean depth of $H = 4000$ m, the correction factors are 0.6 and 0.4, respectively.

Using the correction factor based on bottom station data, only the amplitudes of forced waves can be adequately estimated; applying any correction factors to the superposition of forced and free waves is unjustified, since it is impossible to separate forced and free waves.

For this reason, one should be cautious in identifying the shape of the water surface based on bottom pressure variations in cases where the bottom pressure information contains data on forced waves. Such cases may be events similar to the event of 15.01.2022, or events associated with the passage of cyclones and typhoons over the ocean, accompanied by the excitation of meteotsunamis [22].

The results of solving the problem are of practical importance. Data on bottom pressure variations (7) of deep-ocean stations closest to the disturbance source are used in operational tsunami forecasting. The discrepancy between (7) and (8) may lead to an inadequate assessment of the degree of danger of the predicted tsunami. The question of how adequate the assessment of the amplitude of surface waves based on bottom pressure data is posed in [27].

Expression (7) for bottom pressure variations can be written as

$$\eta_{bott}(-H,t,\xi) = \frac{P_0}{\rho g} \cos Ut\xi + \frac{P_0}{\rho g} \cdot \frac{gH}{U^2 - gH} \cos Ut\xi -$$

$$- \frac{P_0}{\rho g} \cdot \frac{gH}{U^2 - gH} \cos \sqrt{gH} t\xi + Z_0 \cos \sqrt{gH} t\xi \quad (9)$$

The first term in the obtained expression, as expected, coincides with the expression for the forcing pressure (6), the others coincide with the expressions for the shapes of surface, forced and free waves (8). From this representation of the solution for the variations in bottom pressure (9) it follows that it is possible to obtain the true shape of the free surface from the data on the bottom pressure only if the pressure above the free surface is known. Having data on the atmospheric pressure $\eta_{\text{forcing}}(t)$ (6) and subtracting them from the data obtained by the bottom sensors $\eta_{\text{bott}}(t)$ (7), we can obtain the shape of the free surface $\zeta(t)$ (8). The shape of the gravitational component of surface waves can be estimated if we subtract the data on the atmospheric pressure (6) from the data on the bottom pressure (7), multiplied by $(U^2–gH)/U^2$.

Atmospheric pressure above a free surface (in a Lamb wave) can be calculated, for example, using a method based on the algorithms of the express method of operational tsunami forecasting, based on the fundamental principle of reciprocity [24]. To forecast pressure variations at remote points, data on atmospheric pressure of barographs closest to the source of increased pressure can be used.

Conclusion

A solution to the problem of waves in a heavy incompressible fluid layer of constant depth is presented.

A localized short-term increase in pressure above the free surface was specified as a wave source. This results in free waves on the surface and pressure waves on the bottom. The shapes and amplitudes of surface waves and bottom pressure waves (in meters of water column) are the same.

Another wave source was an instantaneous increase in pressure in a localized region and a pressure wave propagating in the atmosphere above the free surface, which occurs as a result of the disintegration of a high-pressure region (the Lamb wave model). Such a source excites free waves on the surface of the layer and on the bottom, traveling at the speed of long waves, and forced waves propagating at the speed of a forcing pressure wave. In the long-wave approximation, the shapes and amplitudes of free waves on the surface and on the bottom are the same. The amplitudes of forced bottom pressure waves are higher than the amplitudes of forced surface waves. To estimate the free surface wave shape based on bottom pressure data, the correction factor gH/U^2 can be applied to the part of the record containing only the forced component. This approach is incorrect for data that include both forced and free components.

Overestimated amplitudes of bottom pressure variations can be the cause of inadequate estimation of the expected tsunami by operational forecasting methods that use bottom pressure data. Despite the uniqueness of events like the event of 15.01.2022, tsunami warning services should probably take this feature into account.

A method is proposed for estimating the free surface shape, including forced (baric) and free (gravity) waves, as well as free wave shapes, based on bottom pressure data.

One of the possible methods for calculating the pressure in the Lamb wave is proposed based on the barometric data of stations closest to the source of high pressure, based on the fundamental reciprocity principle.

References

1. Terry J.P., Goff J., Winspear N., Bongolan V.P., Fisher S. **2022.** Tonga volcanic eruption and tsunami, January 2022: globally the most significant opportunity to observe an explosive and tsunamigenic submarine eruption since AD 1883 Krakatau. *Geoscience Letters*, 9(24). https://doi.org/10.1186/s40562-022-00232-z
2. Solovieva M.S., Padokhinb A.M., Shalimov S.L. **2022.** Mega-eruption of the Hunga Volcano on January 15, 2022: Detection of ionospheric perturbations by VLF and GNSS radio sounding. *JETP Letters*, 116(11): 846–851.
3. Amores A., Monserrat S., Marcos M., Argüeso D., Villalonga J., Jorda G., Gomis D. **2022.** Numerical simulation of atmospheric Lamb waves generated by the 2022 Hunga–Tonga volcanic eruption. *Geophysical Research Letters*, 49, e2022GL098240. https://doi.org/10.1029/2022GL098240
4. Kubota T., Saito T., Nishida K. **2022.** Global fast-traveling tsunamis driven by atmospheric Lamb waves on the 2022 Tonga eruption. *Science*, 377(6601): 91–94. https://doi.org/10.1126/science.abo4364
5. Matoza R.S., Fee D., Assink J.D., Iezzi A.M., Green D.N., Kim K., Toney L., Lecocq T., Krishnamoorthy S., Lalande J.-M. et al. **2022.** Atmospheric waves and global seismoacoustic observations of the January 2022 Hunga eruption, Tonga. *Science*, 377(6601): 95–100. https://doi.org/10.1126/science.abo7063
6. Nosov M.A., Kolesov S.V., Sementsov K.A. **2023.** Interpretation of signals recorded by ocean-bottom pressure gauges during the passage of atmospheric lamb wave on 15 January 2022. *Remote Sensing*, 15(12), 3071. https://doi.org/10.3390/rs15123071
7. Gusman A.R., Roger J., Noble C., Wang X., Power W., Burbidge D. **2022.** The 2022 Hunga Tonga–Hunga Ha'apai volcano air-wave generated tsunami. *Pure and Applied Geophysics*, 179: 3511–3525. https://doi.org/10.1007/s00024-022-03154-1
8. Landau L.D., Lifshitz E.M. **1987.** *Course of theoretical physics*. Vol. 6. 2nd Engl. ed. *Fluid mechanics*. Pergamon Press. (Translation from the Russian).
9. Purkis S.J., Ward S.N., Fitzpatrick N.M., Garvin J.B., Slayback D., Cronin S.J., Palaseanu-Lovejoy M., Dempsey A. **2023.** The 2022 Hunga-Tonga megatsunami: Near-field simulation of a once-in-a-century event.

Science Advances, 9(15). https://doi.org/10.1126/sciadv.adf5493

10. Shrivastava M.N., Sunil A.S., Maurya A.K., Aguilera F., Orrego S., Sunil P.S., Cienfuegos R., Moreno M. **2023.** Tracking tsunami propagation and Island's collapse after the Hunga Tonga Hunga Ha'apai 2022 volcanic eruption from multi-space observations. *Scientific Reports*, 20109. https://doi.org/10.1038/s41598-023-46397-1
11. Nosov M.A., Sementsov K.A., Kolesov S.V., Pryadun V.V. **2022**. Atmospheric Lamb wave manifestation in bottom pressure variatbons. *Moscow University Physics Bulletin*, 77(6): 896–904.
12. Miyashita T., Nishino A., Ho T.-C., Yasuda T., Mori N., Shimura T., Fukui N. **2023.** Multi-scale simulation of subsequent tsunami waves in Japan excited by air pressure waves due to the 2022 Tonga volcanic eruption. *Pure and Applied Geophysics*, 180: 3195–3223. https://doi.org/10.1007/s00024-023-03332-9
13. Tanioka Y., Yamanaka Y., Nakagaki T. **2022.** Characteristics of the deep sea tsunami excited offshore Japan due to the air wave from the 2022 Tonga eruption. *Earth, Planets and Space*, 74(61). https://doi.org/10.1186/s40623-022-01614-5
14. Kubo H., Kubota T., Suzuki W., Aoi S., Sandanbata O., Chikasada N., Ueda H. **2022.** Ocean-wave phenomenon around Japan due to the 2022 Tonga eruption observed by the wide and dense ocean-bottom pressure gauge networks. *Earth, Planets and Space*, 74(104). https://doi.org/10.1186/s40623-022-01663-w
15. Omira R., Ramalho R.S., Kim J., González P.J., Kadri U., Miranda J.M., Carrilho F., Baptista M.A. **2022.** Global Tonga tsunami explained by a fast-moving atmospheric source. *Nature*, 609: 734–740. https://doi.org/10.1038/s41586-022-04926-4
16. Fujii Y., Satake K. **2024.** Modeling the 2022 Tonga eruption tsunami recorded on ocean bottom pressure and tide gauges around the Pacific. *Pure and Applied Geophysics*, 181: 1793–1809. https://doi.org/10.1007/s00024-024-03477-1
17. Ren Z., Higuera P., Liu P.L.-F. **2023.** On tsunami waves induced by atmospheric pressure shock waves after the 2022 Hunga Tonga–Hunga Ha'apai volcano eruption. *Journal of Geophysical Research: Oceans*, 128, e2022JC019166. https://doi.org/10.1029/2022JC019166
18. Heidarzadeh M., Gusman A.R., Ishibe T., Sabeti R., Sepic J. **2022.** Estimating the eruption-induced water displacement source of the 15 January 2022 Tonga volcanic tsunami from tsunami spectra and numerical modeling. *Ocean Engineering*, 261(112165). https://doi.org/10.1016/j.oceaneng.2022.112165
19. Hu G., Li L., Ren Z., Zhang K. **2023.** The characteristics of the 2022 Tonga volcanic tsunami in the Pacific Ocean. *Natural Hazards and Earth System Sciences*, 23(2): 675–691. https://doi.org/10.5194/nhess-23-675-2023
20. Proudman J. **1929.** The effects on the sea of changes in atmospheric pressure. *Geophysical Journal International*, 2: 197–209. https://doi.org/10.1111/j.1365-246X.1929.tb05408.x
21. Monserrat S., Vilibic I., Rabinovich A.B. **2006.** Meteotsunamis: atmospherically induced destructive ocean waves in the tsunami frequency band. *Natural Hazards and Earth System Sciences*, 6(6): 1035–1051. https://doi.org/10.5194/nhess-6-1035-2006
22. Saito T., Kubota T., Chikasada N. Y., Tanaka Y., Sandanbata O. **2021.** Meteorological tsunami generation due to sea-surface pressure change: Threedimensional theory and synthetics of ocean-bottom pressure change. *Journal of Geophysical Research: Oceans*, 126, e2020JC017011. https://doi.org/10.1029/2020JC017011
23. Liu P.L.-F., Higuera P. **2022.** Water waves generated by moving atmospheric pressure: Theoretical analyses with applications to the 2022 Tonga event. *Journal of Fluid Mechanics*, 951(A34). https://doi.org/10.1017/jfm.2022.840
24. Korolev Yu.P. **2011.** An approximate method of short-term tsunami forecast and the hindcasting of some recent events. *Natural Hazards and Earth System Sciences*, 11(11): 3081–3091. https://doi.org/10.5194/nhess-11-3081-2011
25. Lavrent'ev M.A., Shabat B.V. **1973.** [*Methods of complex variable theory*]. Moscow: Nauka, 736 p. (In Russ.).
26. Stoker J.J. **1957.** *Water waves*. New York, London: Interscience Publ. https://doi.org/10.1002/9781118033159
27. Korolev Yu.P., Korolev P.Yu. **2025.** Assessment of the tsunami in the Pacific Ocean caused by the explosion of the Hunga Tonga–Hunga Ha'apai volcano on January 15, 2022, using the express method of operational forecasting. *Geosistemy perehodnykh zon = Geosystems of Transition Zones*, 9(1): 56–65. https://doi.org/10.30730/gtrz.2025.9.1.056-065; https://www.elibrary.ru/kktwzl

About the Author

Korolev, Yury P. (https://orcid.org/0000-0002-7068-7341), Cand. Sci. (Physics and Mathematics), Associate Professor, Leading Researcher, Tsunami Laboratory, Institute of Marine Geology and Geophysics, Far Eastern Branch, Russian Academy of Sciences, Yuzhno-Sakhalinsk, Russia, Yu_P_K@mail.ru

Об авторе

Королёв Юрий Павлович (https://orcid.org/0000-0002-7068-7341), кандидат физико-математических наук, доцент, ведущий научный сотрудник, лаборатория цунами, Институт морской геологии и геофизики ДВО РАН (ИМГиГ ДВО РАН), Южно-Сахалинск, Россия, Yu_P_K@mail.ru

Received 24 April 2025
Accepted 15 June 2025

Поступила 24.04.2025
Принята к публикации 15.06.2025